WO 2004/025384

Rec'd PCT/PTO 08 MAR 2005
PCT/FI2003/000669

10/527026

METHOD AND APPARATUS FOR DETERMINING HYSTERESIS

FIELD OF THE INVENTION

[0001] The invention relates to process automation systems.

BACKGROUND OF THE INVENTION

[0002] In a process automation system, a control loop typically consists of a process, a measurement, a controller and an actual control element, such as a valve, and a related device, such as a valve controller (positioner) and an actuator. An optimal process control depends on how appropriately all these components function.

[0003] In the processing industry, such as the pulp and paper industry, and petroleum refining, petrochemical and chemical industries, various control valves mounted in the plant pipe system control material flows in the process. A material flow may contain any fluid material, such as flowing substances, liquors, fluids, gases or vapours. At its simplest, the control valve may be a manually-controlled mechanical valve. Usually, the valve is provided with a valve controller and an actuator. The valve controller and actuator adjust the position of a control valve according to the control input (e.g. pneumatic or electric control input) received from the process control system.

[0004] Figure 1 exemplifies a functional block diagram of a control valve. A valve controller (i.e. positioner) 10 controls the travel/position (h) of the valve by means of the torque generated by an actuator 11. Position information (h) is provided as feedback from the actuator 11 or valve 12 to an adder arranged at the input of the valve controller. The function of the valve controller is mainly based on an error (e) between an input signal u (control signal from the process) and a feedback position (h). The valve controller 10 minimizes this error by a control algorithm, such as a state or PID algorithm. This control algorithm is tailored for each valve and, if necessary, it can be tuned during the installation or operation. The tuning may include changing gain parameters. It is also feasible to use one or more additional feedbacks in the valve controller 10, such as speed or pressure feedback from the actuator's cylinder, to obtain a more balanced and accurate control function of the valve position.

[0005] Figure 2 illustrates a typical model of a process control loop which controls one control valve 22 and therethrough one material flow in a process. The control valve 22 may be similar to the one shown in Figure 1, for

instance. The process control loop includes a process controller 21, which is provided with a control algorithm, which produces a control signal u for controlling the control valve 22 according to a set point r (which is obtained from a control room computer, for example) and a feedback process variable y. The control algorithm may be any algorithm that is used in control systems, such as PID, PI or P control. The control signal u fed to the control valve 22 controls the valve position and travel and thus the material flow in the process. A desired process variable y is measured by a measurement transmitter 24 and it is compared (block 20) to the set point r of the same process variable to produce an error signal e2, which is fed into the process controller 21. The process controller 21 changes the control signal to minimize the control error 2e. The process control error typically results from changes in a positioning error and process interference.

[0006] The valve and its auxiliary devices often constitute the weak link in the control loop since they are the only moving parts. This movement causes problems, which decrease the capacity of the control loop. To avoid a backlash resulting from mechanical adjustments, the valve, actuator and valve controller/positioner have to be provided with mechanical tolerances that are sufficiently tight. As a result of the backlash, the valve movement does not follow the control signal accurately but deviates from it. The influence of the backlash becomes apparent in particular when the valve control direction and thus the valve's direction of movement are reversed. In that case, the control signal value keeps changing for a while until the measured output signal starts to change noticeably. This is also known as the dead band of control. In addition to the backlash in an actuator or positioner, this phenomenon may result from sticking of the valve or other mechanical factors, such as initial friction. The backlash between mechanical parts naturally increases as the parts wear.

[0007] The backlash and other error factors cause hysteresis between the control of the process device, such as a valve and/or its auxiliary devices, and the measured response. This is illustrated in Figure 3. Straight line 31 illustrates an ideal relation, i.e. characteristic curve, between the control u and the measurement (output) y, such as valve position. The real dependency between the measurement and the control is illustrated by characteristic curve 32. As appears from Figure 3, due to the backlash and any other factors, the upward control (increasing u) has a characteristic curve 32A different from that of the downward control (decreasing u), which has characteristic curve

32B. The difference between the curves represents hysteresis in the control of the process device.

[0008] In some cases, the controllers are provided with automatic backlash compensation, which attempts to take the mechanical non-ideality of the device into account always when the control direction is reversed. This approach is described in US patent 5,742,144, for example. Approach of this kind is good in theory but limited in practice since the backlash and hysteresis vary due to different factors.

[0009] The information on hysteresis and backlash is, however, important to the tuning of the control circuit. It also gives useful information on the condition of the process device, such as a valve and/or its actuator or valve positioner. If hysteresis or backlash increases significantly, service measures can be taken to fix the matter.

[0010] A typical way of detecting the hysteresis or backlash of an actuator is to switch the controller to a manual controlling mode and perform a sequence of step tests. In that case, the actuator is driven to the same position from different directions, in which case any differences between the control and the response due to backlash or hysteresis are found out by means of measurements. Another typical way is to drive the actuator back and forth over the whole control area and to estimate backlash and hysteresis from the measurement results. In the case of a valve actuator, for example, the valve is driven from the closed position to the open position and back to the closed position. A problem associated with these solutions is, however, that they are separate tests that need to be carried out when the process is interrupted or the process device to be examined is bypassed or detached from the process. Similar tests that are performed on the valve positioner are described in IEC (International Electrotechnical Commission) standard 61514, Industrial process control systems: Methods of evaluating the performance of valve positioners with pneumatic outputs, first edition, 2000-04.

[0011] WO 01/11436 discloses a method and an apparatus which statistically determine estimates for one or more process control loop parameters for the device or the control loop that is active in the process control environment. Such parameters include friction, dead band, dead time, vibration or backlash. In the method, one or more signals are always measured in the process control loop when the process control loop is connected to the on-line process control environment. The measured signal is stored as signal data,

after which a number of statistical analyses are carried out on the stored data to determine the desired parameter estimate. An advantage of this solution is that the process device does not need to be removed from process or the control loop bypassed for the test.

[0012] In practice, the on-line determination of hysteresis or backlash is sensitive to process interference as well as inaccurate. Furthermore, it usually requires statistical calculation methods, matrix calculation, mathematical functions, etc.

BRIEF DESCRIPTION OF THE INVENTION

[0013] The object of the invention is to provide a new method and apparatus for determining hysteresis or backlash of a process device in normal run in a process in a relatively simple and accurate manner.

[0014] The object of the invention is achieved by the method and system described in the attached independent claims. Preferred embodiments of the invention are described in the dependent claims. The present invention comprises collecting sample pairs of a signal representing the control of a process device in normal run and a signal representing its output. The signal representing the output can be, for example, a process variable measured in the process. It may also be the set point of a process variable. Each collected pair of control/output samples comprises an average control input and a measured average output, which have been calculated for a predetermined collection period. In an embodiment of the invention, a momentary sample pair is typically taken from the signal representing the control input and from the measured signal representing the output at certain intervals, the interval being preferably in the order of one or more seconds. Such data is often called 'second-level data' or 'seconds data'. As in one embodiment of the invention, an average sample pair is then calculated from the momentary sample pairs (e.g. seconds data) taken during the collection period. The collection period is preferably in the order of one or more minutes, and thus the mean values are often called 'minute mean values'.

[0015] Before calculating characteristic curves, sample pairs suitable for further processing are screened from the collected raw data on the basis of the magnitude and direction of the relative change of the average control input. This screening is of great importance to the accuracy of the method. In normal run, the control circuit continuously controls the process device so

that the control zigzags in both directions. In addition, various kinds of interference occur. By screening suitable sample pairs for further processing on the basis of the magnitude and direction of the relative change of their control input, corner or turning points of the control situation where the control direction changes can be found out. As the control direction changes, the measured output may fall in a vague area due to hysteresis or backlash and thus cause errors in the final calculation of characteristic curves. The screening according to the invention ensures that the samples selected for calculation represent a stable situation either in upward control or in downward control. In an embodiment of the invention, this selection or screening is carried out by calculating the change in the average control input in relation to the average control input of the previous sample pair and the direction of change for each sample pair. If the calculated change is smaller and has occurred in the same direction as the change calculated for the control input of the previous sample pair, the sample pair is selected for further processing; otherwise the sample pair is rejected.

[0016] According to the invention, the sample pairs are also grouped into a first group and a second group according to the direction or sign of the relative change of the average control input of each sample pair. These two groups represent upward control (increasing control value, positive sign) and downward control (decreasing control value, negative sign). The grouping can also take place before, during or after screening. By means of the screened sample pairs of the first and the second group, a first and a second control/output characteristic curve representing ascending and descending control are calculated. Finally, the hysteresis or backlash of the process device is determined on the basis of these characteristic curves as a distance between the first and the second characteristic curve at least at one point. In practice, the calculation of a characteristic curve comprises calculation of at least a few points for each characteristic curve. The determination of hysteresis or backlash means that the distance parallel with the control axis is determined at least at one point determined this way.

[0017] It is important for the operators of an industrial process to know how reliable the hysteresis or backlash obtained in the manner according to the invention is. The reliability varies with different process conditions. In an embodiment of the invention, a reliability value is also calculated for the determined hysteresis or backlash by a function, which includes the following information as parameters

the number Ny1 of average sample pairs belonging to the first, ascending characteristic curve,

the number Ny2 of average sample pairs belonging to the second, descending characteristic curve,

the number Noy1 of sample pairs which belong to the first characteristic curve but are below the second characteristic curve, and

the number Noy2 of sample pairs which belong to the second characteristic curve but are above the first characteristic curve.

[0018] This determination of reliability is based on the fact that the higher the number of the measurement pairs which are used for calculating a certain characteristic curve and are below the second curve, the more unreliable the result is. According to an embodiment of the invention, the reliability value is calculated by function $L=\max(0,1-Noy1/Ny1-Noy2/Ny2)$, in which case L may obtain values from 0 to 1, where L=1 is completely reliable and L=0 is completely unreliable.

BRIEF DESCRIPTION OF DRAWINGS

[0019] In the following, the invention will be described by means of exemplary embodiments, with reference to the accompanying drawings, in which

Figure 1 is a functional block diagram of a control valve,

Figure 2 illustrates a typical model of a process control loop, and

Figure 3 is a graph illustrating a characteristic curve and hysteresis of the control loop,

Figure 4 is a schematic functional block diagram illustrating an algorithm according to an embodiment of the invention for determining hysteresis,

Figure 5 is a data flow chart illustrating the algorithm shown in detection block 40 of Figure 4,

Figure 6A illustrates minute mean values of measurement y,

Figure 6B illustrates minute means values of control u as a function of time,

Figure 7A illustrates characteristic curves formed without the selection according to the invention,

Figure 7B illustrated characteristic curves formed using the selection according to the invention,

Figures 8 and 9 illustrate calculation of hysteresis from the characteristic curves, and

Figure 10 is a graphical presentation where the upper image illustrates control measurement pairs and a calculated characteristic curve, and the lower image illustrates a frequency curve.

DETAILED DESCRIPTION OF THE INVENTION

[0020] The present invention is applicable to all industrial processes and the like. The invention is not limited to any particular process device but it can be applied in various devices that control a process, such as valves, pumps, fans, heat exchangers, etc.

[0021] The exemplary embodiments of the invention will be described using control valves and their auxiliary devices, i.e. valve controllers (positioners) and actuators, as examples.

[0022] Figure 4 is a schematic functional block diagram illustrating an algorithm according to an embodiment of the invention for determining hysteresis. The controller 21 is similar to the PID process controller shown in Figure 2, for instance. It receives as input a set point r of certain type and a measured process variable or another measurement y, which represents the control response or output of the control circuit or process device to be monitored. The controller 21 provides control u, which is fed into the process device, such as a valve controller, an actuator or a positioner. Block 40 represents hysteresis identification according to an embodiment of the invention. In the example of Figure 4, block 40 collects samples from the control signal u and feedback measurement result y. The measurement result may be the measured value of a controllable process variable, such as the measured flow as valve output. Instead of the actual controllable process variable, the measurement result y may be any measured process variable that represents the output and correlates appropriately with the controllable process variable or output. Alternatively, the measurement result may be replaced with a sample representing the set point r. This may be done particularly in cases where the measurement value y has been noted to follow well the set point r. This is the case in the example illustrated in Figure 6A, for instance.

[0023] In an embodiment of the invention, the hysteresis detection block 40 follows the algorithm illustrated by the data flow chart of Figure 5. Block 40 collects second-level sample data y_s and u_s on the process from the

control and measurement signals u and y of the control circuit. In the second-level data, a typical sampling rate is one second or a few seconds, but the invention is not limited to any particular sampling rate. This sample data is processed in different stages and as a result, the calculation provides an estimate for the hysteresis and reliability of the process device or control circuit. The method can be divided into an on-line phase and an off-line phase, but this division is not necessary or it can be performed differently from what is described in this example.

[0024] In the on-line phase, second-level measurements are collected from the process control circuits or from the controller 21 control u and measurement y using another collecting rate, as illustrated in phase 50 in Figure 5. Minute mean values y_m and u_m are calculated from the second-level measurement values in step 51 and these are stored in a database in step 52. The minute mean value means that the mean values are calculated from second-level values collected during a collection period whose duration is approximately one minute or in the order of minutes. The collection period over which the mean value is calculated may vary depending on the application. The controller speed or integration time T_i is typically taken into account in the selection of a mean value period so that the updating period T_p (collecting period) of mean values fulfils the condition $T_I < T_P < 5 \times T_I$. If the controller's integration time T_i were short in relation to the updating period T_P, the controller 21 would have enough time to perform several controls during one period T_P. To give an example, T_I is typically in the order of 10 to 20 seconds in flow control. **[0025]** If the automation system already includes on-line collection for another purpose, by means of which minute mean values, for example, are collected from the control and measurement signals of the control circuit, this information can be utilized as such for the purpose of the present invention. In that case, the existing operations would replace steps 50, 51 and 52 in Figure 5.

[0026] The minute-level (u, y) pairs stored in the on-line phase are retrieved later, for example once a day, once a week or at another interval, from the database for hysteresis calculation. The pairs suitable for hysteresis calculation are selected from the stored minute-level sample pairs u_m and y_m according to a certain procedure (step 53). The pairs are also divided into two groups. Unsuitable pairs are rejected (step 57). Two characteristic curves (54) are calculated from the selected pairs (u_m, y_m) for calculating hysteresis (step

55). The calculation 55 also includes a routine which estimates the reliability of the identified hysteresis.

[0027] In the following, implementation of different phases of the algorithm according to the invention will be described in greater detail by examples.

Calculation of minute mean value

[0028] In an embodiment, the second-level sample pairs (or sample pairs taken at another collecting interval) from the measurement and control signals y and u are collected into buffers $y_{s(i)}$ and $u_{s(i)}$, where i = 1...N (the buffer length is N = 60 when the second/minute calculation is used). The minute-level mean values y_m and u_m or mean values with another storing frequency are calculated on the basis of the seconds data included in the buffers.

Selection and division of pairs of control and measurement

[0029] In an embodiment before calculating a characteristic curve, the pairs of control and measurement samples suitable for hysteresis calculation are selected from the minute-level raw data. The selected sample pairs are further divided into two groups G1 and G2 depending on whether the control circuit control goes upwards (the value of control u increases) or downwards (the value of control u decreases).

[0030] A pair (u_m(j), y_m(j)) of minute mean values is selected for the calculation of characteristic curve if the following two conditions are fulfilled:

$$|\Delta u_m(j)| < |\Delta u_m(j-1)|$$
$$sign(\Delta u_m(j)) = sign(\Delta u_m(j-1))$$

where
u_m(j) is the average control input of sample pair j,
y_m(j) is the average measured output of sample pair j,
j is an integer index,
$\Delta u_m(j) = u_m(j) - u_m(j-1)$
sign(Δu_m) calculates the sign of Δu_m.

[0031] In other words, a pair of measurement and control samples is accepted for hysteresis calculation only if the change Δu_m(j) in the minute

mean value $u_m(j)$ of the control is smaller than the change calculated for the control of the previous sample pair and the change has occurred in the same direction as the previous change.

[0032] The pairs of control and measurement samples are further divided into two groups on the basis of the sign of the control change Δu_m as follows: a sample pair is taken in group G1 when the sign of the change is positive (the control goes upwards) and a sample pair is taken in group G2 when the sign of the change is negative (the control goes downwards).

[0033] The selection and grouping carried out according to the principles of an embodiment of the invention are very essential to the accuracy and reliability of hysteresis determination. This can be examined by means of an example related to flow control. Minute mean values are collected from measurement and control signals of a control circuit for approximately 6.5 hours according to the invention. Figure 6A illustrates minute mean values of measurement y and Figure 6B illustrates minute mean values of control u as a function of time. It can be seen from Figures 6A and 6B that the control signal u zigzags; a phenomenon that may indicate a fault in the operation of an actuator. If pairs of control and measurement samples are formed from the minute mean values shown in Figures 6A and 6B and characteristic curves are drawn without the selection according to the invention, we obtain the result shown in Figure 7A. It can be seen that the pairs of control and measurement samples fill the whole hysteresis area and no clear curves are discernible. Next, we will examine an embodiment according to the invention where the pairs of control and measurement samples suitable for hysteresis calculation are selected and the pairs are divided into two groups. The selected samples pairs are shown in Figure 7B, where the sample pairs of group G1 (control moves upwards) are marked with symbol o and the sample pairs of group G2 (control moves downwards) are marked with symbol x. Furthermore, two characteristic curves -*- have been drawn on the basis of these pairs. It appears from Figures 7A and 7B that the selection and grouping according to the invention transform the vague set of points shown in Figure 7A into two clear characteristic curves, of which one represents upward control (in the positive direction) and the other downward control (in the negative direction). This simple example shows that the selection and grouping have great significance for the accuracy of the method according to the invention.

Calculation of characteristic curves

[0034] In an embodiment after the selection and division, characteristic curves are calculated from the sample pairs in step 54. There are two characteristic curves, both of which are calculated by the same algorithm. The following describes an algorithm for calculating one characteristic curve.

[0035] A control area (value range of control signal) is divided into bins $u_0(1)\ldots u_0(n_{bin})$, in which the values of the characteristic curve are stored as the calculation proceeds. Points $u_0(1)\ldots u_0(n_{bin})$ denote bin locations on the u-axis representing the control input and bin denotes the number of bins. Parameters $y_0(1)\ldots y_0(n_{bin})$ represent the values of output (measurement) y on the y-axis that represents the output. Thus the pairs $u_0(1)/y_0(1),\ldots u_0(n_{bin})/y_0(n_{bin})$ formed by the bin locations and values define a characteristic curve. In addition, the number of sample pairs used in the calculation of each bin value is calculated in counters $nct(1)\ldots nct(n_{bin})$. The number of bins may be any desired one, for example 3, 5 or 10. The bin $u_0(1)\ldots u_0(n_{bin})$ values $y_0(1)\ldots y_0(n_{bin})$ are updated by means of the selected sample pairs using a predetermined weighting function.

[0036] In an embodiment of the invention, two pairs b_n, b_{n-1} are updated according to each sample pair and these pairs are selected so that the following condition is fulfilled:

$$b_n < \frac{u_m - u_{\min}}{u_{\max} - u_{\min}}(n_{bin} - 1) + 1 < b_{n+1}$$

where

u_m is the minute mean value of the control sample pair j,

y_m is the minute mean value of the measurement of sample pair j,

n = 1...bin,

u_{min} and u_{max} are the minimum and the maximum of the control area, respectively.

[0037] After this, the values $y_0(b_n)$ and $y_0(b_{n+1})$ of the selected bins b_n, and b_{n-1} are updated as follows

$$y_0(b_n) = \frac{nct(b_b)y_0(b_n) + w_1 y_m}{nct(b_n) + w_1}$$

$$y_0(b_{n+1}) = \frac{nct(b_{n+1})y_0(b_{n+1}) + w_2 y_m}{nct(b_{n+1}) + w_2}$$

where

$nct(1)...nct(n_{bin})$ each represents the number of updates (con trol/measurement pairs used) of each bin,

w_n and w_{n+1} are weighting coefficients

$$w_n = 1 - \frac{|u_m - u_{min} - (b_n - 1)u_{st}|}{u_{st}}$$

$$w_{n+1} = 1 - \frac{|u_m - u_{min} - (b_{n+1} - 1)u_{st}|}{u_{st}}$$

u_{st} is the distance between bins

$$u_{st} = \frac{u_{max} - u_{min}}{n_{bin} - 1}$$

[0038] Finally, the numbers nct(bn) and nct(bn-1) of updates of the selected bins bn and bn-1 are updated as follows

$nct(b_n) = a*nct(b_n) + w_1$

$nct(b_{n+1}) = a*nct(b_{n+1}) + w_2$

where a is a constant.

[0039] Constant a is a 'forgetting factor'. Usually a = 1 is selected as the value of forgetting factor a, which means that forgetting is not in use. However, in continuous monitoring it may be advantageous to select a value a < 1, typically a = 0.9-0.9999. This value means that a newer measurement obtains a higher weighting coefficient in bin updating than an older measurement. Thus the influence of older measurements disappears gradually, i.e. they are "forgot".

[0040] It should be noted that the method of calculating curves described here is only one feasible calculation method; yet it improves the accuracy of calculation. Based on the above description, it is obvious to a person skilled in the art that other methods can also be used for calculating characteristic curves on the basis of the minute mean values selected and grouped according to the invention.

Hysteresis calculation

[0041] The calculation 54 of characteristic curve produces two characteristic curves (or points of characteristic curves). One of the curves represents ascending control and the other one descending control. Hysteresis is calculated as a distance between the characteristic curves at one or more points in the direction of the control axis (u-axis). The selection of the number and location of the points preferably depends on how control has moved during a collection period.

[0042] Hysteresis calculation will be exemplified at one point with reference to Figure 8. In the example, two points are known in one characteristic curve (u_A, y_A) and (u_C, y_C) and one point in the other characteristic curve (u_B, y_B) so that $y_A > y_B > y_C$. The distance between the characteristic curves in direction h of the u-axis is

$$h = u_A - u_B + \frac{(u_C - u_A)(y_B - y_A)}{(y_C - y_A)}$$

[0043] Figure 9 illustrates two characteristic curves where hysteresis has been calculated at several points. The values of the control axis (u-axis) of the characteristic curves shown in Figure 9 describe the control's relative share (%) of the whole control area (100%).

Example

[0044] The calculation of the characteristic curve in the embodiment described above will be described here by an example. For the sake of clarity, the example describes only the calculation of one characteristic curve. In practice, the hysteresis calculation updates two characteristic curves according to the same principle.

[0045] Parameters used in the example

Number of bins: $n_{bin} = 4$
Control min: $u_{min} = 0$
Control max: $u_{max} = 30$
Thus the distance between the bins is $u_{st} = 10$.

[0046] Before updating, the vectors describing the characteristic curve do not actually include any information. The vectors are

- Measurements y0 of characteristic curve
- Controls u0 of characteristic curve
- Frequency information (number of hits) nct on characteristic curve

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y0  = [0   0   0   0]
u0  = [0  10  20  30]
nct = [0   0   0   0]
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[0047] The following pair of control and measurement is added to the characteristic curve

$u_m = 12$
$y_m = 3$

First, bins are selected according to the following formula

$$b_1 < \frac{u_m - u_{\min}}{u_{\max} - u_{\min}}(n_{bin} - 1) + 1 < b_2$$

$$\frac{u_m - u_{\min}}{u_{\max} - u_{\min}}(n_{bin} - 1) + 1 = \frac{12-0}{30-0}(4-1)+1 = 2.2 \Rightarrow b_1 = 2, b_2 = 3$$

Weighting coefficients are calculated

$$w_1 = 1 - \frac{|u_m - u_{\min} - (b_1 - 1)u_{st}|}{u_{st}} = 1 - \frac{|12 - 0 - (2-1)\times 10|}{10} = 0.8$$

[0048] Measurement vectors y0 are updated according to the following formula

$$y_0(b_1) = \frac{nct(b_1)y_0(b_1) + w_1 y_m}{nct(b_1) + w_1} = \frac{0 + 0.8 \times 3}{0 + 0.8} = 3$$

[0049] Measurement vectors y0, bin 3 and bins 2 and 3 of the control vector u0 are updated in the same manner.

Vector nct is updated

$$nct(2) = a*nct(2) + w_1 = 1*0 + 0.8 = 0.8$$

and the same is performed on bin 3. Forgetting factor a=1, which means that forgetting is not in use.

[0050] After the first pair of control and measurement, the vectors of the characteristic curve are:

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y0  = [0   3    3    0]
u0  = [0   12   12   30]
nct =[0 0.8 0.2   0]
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[0051] Next, the characteristic curve is updated using the following pair of control and measurement

$u_m = 24$
$y_m = 5$

[0052] Now the control is between bins 3 and 4 ($b_1 = 3$, $b_2 = 4$) so that it is closer to bin 3 (weighting coefficient $w_1 = 0.6$) than to bin 4 (weighting coefficient $w_2 = 0.4$).

The measurement vector is updated as follows

$$y_0(b_1) = \frac{nct(b_1)y_0(b_1) + w_1 y_m}{nct(b_1) + w_1} = y_0(3) = \frac{0.2\times 3 + 0.6\times 5}{0.2 + 0.6} = 4.5$$

The same formula is used in updating the control

$$u_0(3) = \frac{0.2\times 12 + 0.6\times 24}{0.2 + 0.6} = 21$$

vector nct is updated,

nct(3) = nct(3) + w1 = 0.2 + 0.6 = 0.8

y0(4), u0(4), and nct(4) are updated as described above (the bin is empty before updating). Weighting coefficient w2 = 0.4.

[0053] After the second point, the vectors for the characteristic curve are

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y0  = [0   3 4.5  5]
u0  = [0  12  21  24]
nct = [0 0.8 0.8 0.4]
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The following pair is still added

$u_m = 15$
$y_m = 4$

The result is

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y0  = [0    3.3846    4.3077    5.0000]
u0  = [0   13.1538   18.6923   24.0000]
nct = [0    1.3000    1.3000    0.4000]
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[0054] Assume further that the calculation has been continued by adding 97 pairs of measurement and control such that the control is a randomly selected integer between 0 to 30 and the measurement is the square root of this rounded to the nearest integer.

[0055] The upper image in Figure 10 illustrates the pairs (x) of control and measurement used in the example and the calculated characteristic curve (—*—) y0 vs. u0. The lower image illustrates the frequency curve nct vs. u0 (—*—).

[0056] In all the embodiments described above and in other variations of the invention, the measurement y can be replaced with the set point r or a signal representing it.

[0057] The description and the related figures are only intended to illustrate the principles of the present invention. Various embodiments, variations and modifications are obvious to a person skilled in the art on the basis of this description. The present invention is not limited by the examples but it may be modified within the scope and spirit of the attached claims.